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This note is about Kyle (1985) that expands on Prof. Brunnermeier's slides from class. For the truly interested for more details in this subject, I highly recommend Prof. Brunnermeier's book, "Asset Pricing Under Asymmetric Information." This book summarizes very concisely a huge literature about the very general question: how do prices impound information, and why do we observe so much trade in markets? This discussion is loosely based on that book and Prof. Brunnermeier's slides.

1 Quickly...Some Background

Lots of people have asked me, what are we supposed to learn from Kyle (1985)? The Kyle (1985) model is deliberately esoteric - it doesn't give us a neat formula for computing prices, like the Black-Scholes formula, or CAPM. So what are we trying to learn? Here is some quick background.

One of the central questions in financial economics is, why do we observe so much trade in markets, and how does information become impounded in prices? This issue was highly discussed in the early 1980's and continues to be revisited today. The most intuitive answer is that agents trade on the basis of differing information. However, a series of "no-trade theorems" shows that this is not sufficient to guarantee trade. In fact, under certain conditions, *no trade will occur*.

One of the basic tools used to analyze these problems is the rational expectations equilibrium (REE) framework we saw in class, along with the following insight from Aumann (1976). Rational agents cannot agree to disagree about the probability of a given event, if the following are true: 1) we all use Bayes' rule to update our beliefs based on new information (projection theorem), 2) we begin with a common prior (we start with the same beliefs), and 3) rationality of players is common knowledge (I know that you know that I know that you know that...I am rational, ad infinitum.) The intuition behind Aumann's argument is simple: if you have a

different assessment about the likelihood of some event v than I do, the only possible conclusion for me is that this is because you have observed some information that I have yet to impound in my beliefs, and therefore I will update my beliefs to reflect this accordingly. As a final result, *we have the same posterior beliefs, even though we have observed different information.*

Applied to a trading game, these observations (loosely) yield the following result¹. Suppose there is one asset which pays an uncertain payoff v tomorrow, and that you and I have private information about what v will be. Based on my information, I will have some assessment \hat{v} about the fair value of v . Suppose you observe some piece of information which leads to your assessment being $\tilde{v} \neq \hat{v}$. If you offer to trade with me at \tilde{v} , I will naturally conclude that you have observed some piece of information that I have not, and, since you would only trade with me if you profit, I would be foolish to trade with you.

2 Where to go from here?

This line of reasoning is extremely powerful and difficult to overcome. Economists have come to adopt a trick of introducing "noise" traders. (For more on the concept of "noise," see Black (1986)). In addition, by introducing more realistic elements of markets, so-called "market microstructure" elements, we can better understand how prices impound information in a more realistic setting (at a slight loss of generality). The Grossman (1976) REE derived demand and prices under a CARA/Gaussian framework, and we saw that prices reveal a sufficient statistic of information (the mean signal) in this setting, leading to the famous Grossman-Stiglitz paradox.

Two other powerful contributions have been made by Glosten and Milgrom (1985) and Kyle (1985). In Glosten and Milgrom (1985), an uninformed market maker sets prices before an informed trader submits his market order. We saw that given this "adverse selection" problem, market makers use conditional expectations to set bid and ask prices, and that a certain amount of "liquidity" or "noise" trading was required to avoid a market breakdown. Kyle (1985) is a complementary contribution in which the informed trader moves first, rather than the market maker.

¹There are infinite numbers of subtleties with this line of reasoning, about which there have been written hundreds of scholarly articles, but I won't get into that here (see Prof. Brunnermeier's book for a nice survey).

Why is this interesting? Recall that the basic problem an informed trader faces is to trade profitably; in order to do this, he *must trade without revealing his information in the price*. Why? Well, this is why I wrote that stuff in the Background section: if the price will always reveal your information, no one will trade with you, and you can't earn a profit. Kyle (1985) asks the question: suppose I have some private information, and suppose I know there are some liquidity traders out there. *How do I optimally trade to earn the most money off my information?*

2.1 Solving Kyle (1985)

Suppose there are two dates, $t = 0, 1$ and there is one asset which returns $v \sim N(p_0, \Sigma_0)$ tomorrow. There are three types of traders, all risk-neutral. First, there are "insiders." For simplicity, suppose they know v exactly, and face the problem of deciding how much x of the asset to trade to maximize expected profit, $E[(v - p)x|v]$. Second, there are noise traders, who submit market orders randomly; their total demand is $u \sim N(0, \sigma_u^2)$. There is a market maker, who sets a price p at which the total order flow $X = x + u$ is executed, after he observes X . The price is set competitively, i.e. $p = E[v|x + u]$. You can think of this last assumption as there being lots of market makers out there who compete until the price is equal to its conditional expectation.

The timing is as follows. At $t = 0$, insiders and liquidity traders move first, and submit their market orders. Then (still in $t = 0$, but after the insiders/liquidity traders have moved), the market maker observes the order flow x and sets the execution price p . At $t = 1$, all profits are realized.

The trick in these types of problems is to conjecture an equilibrium and verify it. It turns out that the following conjecture works in this simple model. Conjecture that insiders trade according to $x = \alpha + \beta v$ and that the price will end up equal to $p = \mu + \lambda(x + u)$. In order to verify that this works, we simply need to solve for $\{\alpha, \beta, \mu, \lambda\}$ and hope that x and p do indeed turn out to have the functional form required. Two steps are required.

Step 1: Use optimal demand equations to obtain a system that defines $\{\alpha, \beta, \mu, \lambda\}$.

Let's begin with the informed trader, who knows v . Suppose he knows that the market maker

will set price $p = \mu + \lambda(x + u)$. Then his problem is

$$\begin{aligned}\max_x E[(v - p) | v] x &= \max_x E[(v - \mu - \lambda(x + u)) | v] x \\ &= \max_x [x(v - \mu - \lambda(x + u))]\end{aligned}$$

This gives FOC,

$$x = -\frac{\mu}{2\lambda} + \frac{1}{2\lambda}v$$

with SOC, $\lambda > 0$ (which must be verified later). Going back to our conjecture that $x = \alpha + \beta v$, we see that

$$\alpha = \frac{-\mu}{2\lambda}, \beta = \frac{1}{2\lambda}$$

Now let's go to the market-maker. The market maker observes $X = x + u$ and sets price $p = E[v|X]$. Applying the projection theorem and using $x = \alpha + \beta v$ gives us

$$\begin{aligned}p &= E[v] + \frac{Cov[v, x + u]}{Var[x + u]} \{x + u - E[x + u]\} \\ &= p_0 + \frac{\beta \Sigma_0}{\beta^2 \Sigma_0 + \sigma_u^2} \{x + u - \alpha - \beta E[v]\}\end{aligned}$$

(Note that there is a small typo in Markus's slide, it should be $\{x + u - \alpha - \beta E[v]\}$, not $\{x + u - \alpha + \beta E[v]\}$. Re-arranging with $p = \mu + \lambda(x + u)$ gives us

$$\mu = p_0 - \lambda\alpha - \lambda\beta E[v], \lambda = \frac{\beta \Sigma_0}{\beta^2 \Sigma_0 + \sigma_u^2}$$

Step 2: Solve the system. This is just some math. From the market maker's problem, using α and β from above, we get that

$$\begin{aligned}\mu &= p_0 + \frac{\mu}{2} - \frac{1}{2}E[v] \\ &= \frac{1}{2}p_0 + \frac{\mu}{2} \\ &= p_0\end{aligned}$$

Which gives us μ . To get λ , note

$$\lambda = \frac{\beta \Sigma_0}{\beta^2 \Sigma_0 + \sigma_u^2} = \frac{\frac{1}{2\lambda} \Sigma_0}{\frac{1}{(2\lambda)^2} \Sigma_0 + \sigma_u^2}$$

Re-arranging² yields

$$\lambda = \frac{1}{2} \sqrt{\frac{\Sigma_0}{\sigma_u^2}}$$

Note this guarantees that the SOC $\lambda > 0$ is satisfied in the insider's problem.

Finally,

$$\alpha = -\frac{p_0}{\sqrt{\left(\frac{\Sigma_0}{\sigma_u^2}\right)}}, \beta = \frac{1}{\sqrt{\frac{\Sigma_0}{\sigma_u^2}}}$$

2.2 What do we learn?

Note that β is the insider's sensitivity to his information. When the market is very liquid (in the sense of high σ_u^2), the insider makes larger trades. The liquidity traders essentially allow the insider to "camouflage" his information. From the market maker's perspective, the order flow becomes a weak signal of the insider's information, and so the price is relatively insensitive to this order flow; that is, λ is low. But the market maker is compensated (on average) for his losses to the insider by making money off the noise traders (on average).

In the dynamic model, it becomes optimal for the insider to "space out" his trades so as to optimally hide his information. It is not worth it to trade too aggressively based on your information, since then the information becomes impounded in the price too quickly and you

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$$\begin{aligned} \lambda \left(\frac{1}{(2\lambda)^2} \Sigma_0 + \sigma_u^2 \right) &= \frac{1}{2\lambda} \Sigma_0 \\ \frac{1}{4\lambda} \Sigma_0 + \lambda \sigma_u^2 &= \frac{1}{2\lambda} \Sigma_0 \\ \frac{1}{4} \Sigma_0 + \lambda^2 \sigma_u^2 &= \frac{1}{2} \Sigma_0 \\ \lambda^2 \sigma_u^2 &= \frac{1}{4} \Sigma_0 \\ \lambda &= \frac{1}{2} \sqrt{\frac{\Sigma_0}{\sigma_u^2}} \end{aligned}$$

lose profit opportunity.

Note that this was all achieved in a risk-neutral setup, which may seem like a technical assumption, but is quite neat here. Insiders with information limit their trading even when they are risk neutral; usually, in the absence of frictions, a risk-neutral trader would trade "infinity" if his belief about the value of an asset is different than its price.

Finally, note that *the price is a martingale* from the market maker's perspective. This is a simple consequence of the fact that the market maker is learning "optimally" using Bayes's rule: because the price today reflects the market maker's best guess about the asset's value conditional on the information that he observes, any subsequent movements in the price must be unexpected. That is, all movements in the price are innovations to the market maker: $E[p] = p_0$, since $E[\lambda(\alpha + \beta v) + \lambda u] = 0$ (you can verify this).

References

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